M(atrix)-Theory in Various Dimensions

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Abstract

We demonstrate the precise numerical correspondence between long range scattering of supergravitons and membranes in supergravity in the infinite momentum frame and in M(atrix)-Theory, both in 11 dimensions and for toroidal compactifications. We also identify wrapped membranes in terms of topological invariants of the vector bundles associated to the field theory description of compactified M(atrix)-Theory. We use these results to check the realization of T-duality in M(atrix)-Theory.

1. Introduction

In the past few months, a great deal of evidence has been accumulated in favor of the matrix model description of M-Theory proposed by Banks, Fischler, Shenker, and Susskind [1]. This M(atrix)-Theory, as it has come to be known, is the large N limit of the maximally supersymmetric quantum mechanics of $U(N)$ matrices. According to this picture, D0-branes are partons whose bound states represent the 11-dimensional graviton supermultiplet [2,3].

M(atrix)-Theory is formulated in the infinite momentum frame in a compact 11-dimension, so that all modes with negative or vanishing p_{11} component[1] can be integrated out, leaving only the D0-brane bound states for which

$$p_{11} = \frac{N}{r}, \tag{1.1}$$

where r is the radius of the 11-dimension and $N > 0$ labels the number of D0-branes in the state. The uncompactified infinite momentum limit is defined by taking $N, R \to \infty$, $N/R \to \infty$. The decoupling of anti-D0-branes is crucial in avoiding the tachyonic divergence that appears in brane-antibrane interactions at distances of order the string scale [4]. The action of the theory is simply the world-volume effective action generated by open strings stretching between N D0-branes, which can be obtained from the dimensional reduction of $\mathcal{N} = 1$, $D = 9 + 1$ SYM down to $0 + 1$ dimensions [5,6,7]. In [1], the authors demonstrated that the potential between two D0-branes in M(atrix)-Theory is the same as that between 11-dimensional supergravitons, in agreement with the worldsheet calculations of [8,9]. We carefully reproduce this potential in 11-dimensional supergravity in the infinite momentum frame and in M(atrix)-Theory and find exact numerical agreement within the conventions we present.

Toroidal compactification of M(atrix)-Theory was also explained in [1] and elucidated in [10]: M(atrix)-Theory on T^k is equivalent to $k + 1$-dimensional supersymmetric Yang-Mills theory (SYM) on the dual torus. In particular, M(atrix)-Theory compactified on T^3 was considered in [11,12,13], where it was shown that Type II T-duality is guaranteed by the S-duality of the $\mathcal{N} = 4$, $D = 4$ SYM that describes the system. We consider the potential between D0-branes when we toroidally compactify each of the spatial dimensions and again find exact agreement between M(atrix)-Theory and supergravity, including the correct logarithmic potential of 4-dimensional gravity in the infinite momentum frame.

Using a construction first obtained in [14], the authors of [1] showed that M(atrix)-Theory contains supermembranes. In [15,16], the scattering of gravitons and membranes

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[1]We follow [1] in using p_{11} to denote the light-cone momentum $p_+ = p_0 + p_{D-1}$.

and membranes and anti-membranes was considered, where it was shown that M(atrix)-Theory produces the correct potentials of supergravity, up to an ambiguity owing to the degeneracy of states on the membrane world-volume. We consider toroidal compactifications with membranes wrapped on 2-cycles and find that their contribution can be calculated using the first Chern class of the $U(N)$ vector bundle of the SYM theory. We again find total agreement between M(atrix)-Theory and supergravity. We also find an explicit realization of T-duality in the description of wrapped membrane states that can be identified. Furthermore, we expect that wrapped longitudinal 5-branes in M(atrix)-Theory [17,12,18] may be discussed in terms of the second Chern class of the bundle, an observation that is evident in the instanton description of [18].

2. Graviton-Graviton Scattering in Eleven-Dimensional Supergravity

Since M(atrix)-Theory is formulated in the infinite p_{11} frame, we want to compute the scattering amplitude in 11-dimensional supergravity for two gravitons in the infinite momentum frame with zero energy and p_{11} transfer. At tree level, there is no contribution from intermediate gravitinos or antisymmetric tensors, so the calculation is simply that of linearized gravity in D=11 dimensions. A succinct expression for the tree level amplitude may be obtained with the realization that the field theoretic amplitude is easily extracted from the zero Regge slope limit of the Type II superstring four graviton amplitude [19]. A brief discussion of our conventions is given in the Appendix. Since this amplitude contains approximately 150 terms, we will consider the case of immediate interest, in which the gravitons have momenta orthogonal to their polarizations, which was studied in [9,1]. In this case, the amplitude is simply

$$A = -\frac{\kappa^2}{4} \frac{p_{11}^{(1)} p_{11}^{(2)}}{\left(p_\perp^{(1)} - p_\perp^{(4)}\right)^2} \left(\frac{p_\perp^{(1)}}{p_{11}^{(1)}} - \frac{p_\perp^{(2)}}{p_{11}^{(2)}}\right)^4 \tag{2.1}$$

We note that this is precisely what one would obtain from the prescription

$$A = -\kappa^2 \frac{K^{(1)} K^{(2)}}{q_\perp^2}, \tag{2.2}$$

where $K^{(i)}$ denotes the relative kinetic energy in the galilean form appropriate to the infinite momentum frame.

We can obtain the effective graviton-graviton potential from this scattering amplitude by a Fourier transform over the dimensions transverse to the light cone

$$V(R) = \frac{1}{2\pi r} \int \frac{d^9 q_\perp}{(2\pi)^9} A, \tag{2.3}$$

where $1/r$ is the "quantum" of p_{11}. In $d > 2$ dimensions, the scalar Green's function is

$$G_d(x) = \frac{1}{(d-2)\Omega_{d-1}|x|^{d-2}}, \tag{2.4}$$

where Ω_k is the surface area of the k-sphere (here, of course, $d = 9$). Using (A.7) we find the Newton law

$$\begin{aligned} V(R) &= -\frac{15}{16} \frac{r^2}{T_A^3} \frac{p_{11}^{(1)} p_{11}^{(2)}}{|R|^7} \left(\frac{p_\perp^{(1)}}{p_{11}^{(1)}} - \frac{p_\perp^{(2)}}{p_{11}^{(2)}} \right)^4 \\ &= -\frac{15}{16} \frac{N_1 N_2}{T_A^3} \frac{v^4}{|R|^7}, \end{aligned} \tag{2.5}$$

where in the last expression we have used the definitions appropriate to the matrix model,

$$p_{11} = \frac{N}{r}, \qquad v = \Delta\left(\frac{p_\perp}{p_{11}}\right). \tag{2.6}$$

If we compactify on a k-torus T^k, we instead find that

$$V(R) = -\frac{\pi^4}{2\text{Vol}(T^k)} \frac{N_1 N_2 v^4}{T_A^3} \cdot \begin{cases} \left((7-k)\Omega_{8-k}|R|^{7-k}\right)^{-1} & \text{for } k < 7, \\ -\frac{1}{4\pi} \ln R^2 & \text{if } k = 7. \end{cases} \tag{2.7}$$

3. Scattering of D0-branes in M(atrix)-Theory

From considering the tensions of the open strings which stretch between D0-branes one obtains the dimensionally correct M(atrix)-Theory Lagrangian

$$\mathcal{L} = \text{Tr}\left[\frac{1}{2r}(D_t X_i)^2 - \frac{T_A^2}{4r}[X_i, X_j]^2 - \theta^T D_t \theta - \frac{T_A^2}{4}\theta^T \gamma_i[\theta, X_i]. \right], \tag{3.1}$$

where $D_t = \partial_t + iT_A A_0$.

We proceed to quantize this theory in the covariant background field gauge with

$$D_{\mu B} A_\mu = 0, \tag{3.2}$$

where B_0 and the B_α are zero for A_0 and the compact X_α and $B_i = R_i$ for the Higgs components, corresponding to the position vectors of the D0-branes. The advantage of using

this background gauge is that one can directly check that $(F_{0i})^2$ vanishes at one-loop order. Therefore there is no need for finite renormalizations, even though we are not using a gauge that is manifestly supersymmetric. Although we can do a supergraph calculation that respects some of the supersymmetries (since (3.1) may be obtained from the dimensional reduction of $\mathcal{N} = 4$, $D = 4$ SYM to quantum mechanics), such a method does not extend to calculations for M(atrix)-Theory compactifications, since there is no guarantee of a superfield formalism in more than four dimensions. In the background field method all of the components of the gauge field, as well as the ghost fields, couple in the same way to the background field. Therefore we can still benefit from supersymmetry and algebraic cancellations between bosons and fermions are still possible before doing the loop integrals. At the one-loop level, we have results that are free from infinities.

As a simplification, we will consider the interaction of 2 D0-branes, for which the gauge group is $U(2)$. For N D0-branes interacting with N' D0-branes, the group would be $U(N + N')$, but we would get the same answer multiplied by a factor NN' to account for the degeneracy of the off diagonal matrix elements that one integrates out. Additionally, we factor out the irrelevant center of mass motion and actually consider $SU(2)$ matrices. The one-loop effective 4-point function obtained in this way is

$$A = -6 \int \frac{dw}{2\pi} \frac{(T_A v)^4}{(w^2 + T_A^2 R^2)^4} \tag{3.3}$$

which is readily integrated to yield

$$A = -\frac{15}{16} \frac{1}{T_A^3} \frac{v^4}{R^7}, \tag{3.4}$$

which matches (2.5) exactly.

When compactifying on T^k one must consider the $k + 1$ SYM on the dual torus [10]. In particular, this means that v is taken to be the relative velocity in lower dimensions, which corresponds to the motion of the Higgs moduli in the toroidal theory. Moreover, the integral above becomes

$$A = -6 \sum_{n_i} \int \frac{dw}{2\pi} \frac{(T_A v)^4}{(w^2 + T_A^2 R^2 + T_A^2 (n_i e_i)^2)^4}, \tag{3.5}$$

where the e_i are the dual vectors (momentum labels) of the dual torus, which are just the lattice vectors of the original torus. At large R, we can turn the sums into integrals to obtain

$$A = -6 \int \frac{dw}{2\pi} \frac{dx_1 \dots dx_k}{\text{Vol}(T^k)} \frac{(T_A)^{4-k} v^4}{(w^2 + T_A^2 R^2 + x^2)^4}. \tag{3.6}$$

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After integrating this in spherical coordinates, we obtain

$$A = -\frac{1}{\text{Vol}(T^k)} \frac{v^4}{T_A^3} \frac{\pi^{\frac{k-1}{2}} \Gamma\left(\frac{7-k}{2}\right)}{4R^{7-k}}. \tag{3.7}$$

Noting that

$$\Gamma\left(\frac{7-k}{2}\right) = \frac{4\pi^{\frac{9-k}{2}}}{(7-k)\Omega_{8-k}}, \tag{3.8}$$

we recover precisely (2.7) for $k < 7$. More care must be taken when $k = 7$, since the integral obtained diverges logarithmically. We take

$$\lim_{k \to 7} \frac{\Gamma\left(\frac{7-k}{2}\right)}{R^{k-7}} \sim \ln\Lambda - \ln R^2 \tag{3.9}$$

and recover (2.7) for $k = 7$.

One comment about divergences is in order. While it is true that the covariant gauge (3.2) has allowed the maximal supersymmetry of M(atrix)-Theory to work its miracles and thereby maintain zero β-function in gauge theories in as many as $7 + 1$ dimensions, we still have a divergence in (3.9), which we chose to regulate with the UV cutoff Λ. This cutoff has an obvious explanation once we recall that the SYM theory is formulated on the *dual* torus, so that UV scales are exchanged with IR scales. Therefore Λ is simply the cutoff associated to the IR divergence of the supergravity logarithmic potential in (2.7). Since we consider graviton-graviton scattering at zero p_{11} transfer, this IR scale is $1/r$.

4. Wrapped Membranes in M(atrix)-Theory

We would now like to consider M(atrix)-Theory on a torus $\mathcal{T}$ and calculate the scattering of the D0-branes obtained by wrapping supermembranes around 2-cycles in $H_2(\mathcal{T})$. The wrapped membranes should correspond to different topological sectors of the dimensionally reduced SYM that described the compactification. In particular, magnetic fluxes should provide the invariants needed to classify such configurations. A previous discussion of fluxes and wrapped membranes is contained in [12]. One expects to get results that agree with T-duality between the Type II theories. We will show that this is the case.

Let us first consider toroidal compactification to 9 dimensions, namely when $\mathcal{T} = T^2$, which is the smallest manifold upon which we can completely wrap the membrane. Of interest to us are solutions to the Yang-Mills equations which are time-independent and that preserve half of the supersymmetries, *i.e.* which are BPS saturated states. Therefore we will

consider a bound state of N D0-branes which gives rise to a non-trivial background which preserves the full rotation group. In particular, we will take all of the Higgs fields to vanish.

Following Atiyah and Bott [20], we will have a pure Yang-Mills connection on the torus. Part of the invariants are the eigenvalues of the $U(N)$ gauge field strength, which are constant along the surface. This means that the $U(N)$ bundle splits into $U(N_1)\otimes U(N_2)\otimes\cdots\otimes U(N_k)$, where each of the $U(N_i)$ represent collections of N_i degenerate eigenvalues. Moreover, each eigenvalue is quantized as given by its first Chern class, $c_1(U(N_i))$.

To obtain configurations that only break half of the supersymmetries, all of the eigenvalues have to be the same. Under these conditions, all of the curvature is contained in the $U(1)$ factor of the $U(N) \sim SU(N) \times U(1)$ splitting [18]. This means that the contribution of the $SU(N)$ factor comes in the form of Wilson lines; there is a continuum of such configurations. However, the fact that these are connected means that, in the quantum formalism, they correspond to zero modes of the gauge fields. A proper quantization of these zero modes should provide momentum quantization in the compact directions.

We would now like to use this topological information to compute the potential between a D0-brane and a wrapped membrane, described above as a collection of D0-branes. The standard procedure [15,16] is to consider a D0-brane at a large distance R and to integrate out the off-diagonal modes corresponding to open strings which connect the D0-brane to the bound state. As these off-diagonal modes are charged under the $U(1)$, they will generate an effective potential for the remaining light $U(1)$ degrees of freedom. By pure dimensional analysis, this is proportional to

$$N\frac{F^4}{R^5},\tag{4.1}$$

where F denotes the curvature of the $U(1)$ bundle. A more detailed calculation shows the full result to be equal to

$$V(R) = B\frac{(F_{\mu\nu})^4}{R^5},\tag{4.2}$$

where B is the coefficient of v^4/R^5 that appears in (2.7). However, we note that the field strength appearing in the above equation does not have the canonical normalization of gauge theory that we are considering. The correct normalization is fixed by $F_{\mu\nu} = (F_{\mu\nu})_{\text{can.}}/T_A$.

We note that there are contributions to the gravitational potential (4.2) even at zero velocity. As $F = nF_0$, where $[F/2\pi] = c_1(U(1))$, is quantized, for $N = 1$ we therefore interpret the states with different n as a membrane wrapped n times around a 2-cycle $c \in H_2(\mathcal{T})$, with a single 0-brane attached to it[2]. For a given N, we will interpret this state

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[2]We note that this expression for the winding number agrees with that given as equation (8.12) of [1].

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as a membrane wrapped Nn times around the 2-cycle, with N D0-branes attached to it. As the SYM is formulated on the dual torus $\widetilde{\mathcal{T}}$, we must consider flux quantization on the dual 2-cycle, $\tilde{c} \in H_2(\widetilde{\mathcal{T}})$. This quantization condition is $FA_{\tilde{c}} = 2\pi n$, where $A_{\tilde{c}}$ is the area of $\tilde{c}$. We can therefore write the potential as

$$V(R) = BN \frac{1}{R^5} \left(\frac{2\pi n}{T_A A_{\tilde{c}}} \right)^4. \tag{4.3}$$

From the form of the potential (4.3), the n dependence suggests that it can be interpreted as a momentum label for a graviton. Therefore, when F can be made small, this momentum becomes a continuum and one should be able to describe it as the opening of a new dimension. Since the minimum value of F scales with the inverse of the area of the 2-cycle and the energy is proportional to $F^2 A_{\tilde{c}} \sim 1/A_{\tilde{c}}$, this occurs for only a very moderate cost in energy. As the area, A_c, of the 2-cycle that gives the Kaluza-Klein description is proportional to the reciprocal of $A_{\tilde{c}}$, this new dimension becomes important in the effective supergravity field theory when the size of $\mathcal{T}$ shrinks. This is exactly what happens when wrapped strings become light when considering T-duality of Type IIA string theory. We recall that these strings result from wrapping the membrane along the 11-direction from the M-Theory point of view, so the states that become light are membranes wrapped around longitudinal 2-cycles of $\mathcal{T} \times S^1$. In this case, our 2-cycle doesn't correspond to a membrane wrapped along the 11-coordinate, since this dimension is taken to be large in M(atrix)-Theory, but if one rotates the different 2-cycles to make them match, one obtains agreement with Type II results.

Let us explicitly verify the above claim. By expressing $A_{\tilde{c}}$ in terms of the area, A_c, of its dual 2-cycle in $\mathcal{T}$ and using (A.6), we find that (4.3) can be written as

$$V(R) = BN \frac{1}{R^5} \left(\frac{nT^{(2)} A_c}{1/r} \right)^4. \tag{4.4}$$

Comparing this with the M(atrix)-Theory identifications (2.6), we identify

$$p_\perp = nNT^{(2)} A_c, \tag{4.5}$$

which is precisely the value expected for a wrapped membrane, as we described before. Moreover, the wrapped membrane has the appropriate quantum numbers of a graviton, as the $U(1)$ is totally decoupled as it corresponds to the center of mass motion. The 8 fermionic zero-modes furnish the 256-fold degeneracy, as required by supergravity.

Brane-antibrane scattering can now be straightforwardly analyzed by reversing the orientation of F on a second block of branes. As the non-diagonal blocks are charged with opposite signs under both $U(1)$s, the net result will be proportional to

$$NN' \frac{(F - F')^4}{R^5}. \tag{4.6}$$

Note that when the field strengths match, we have the case of parallel branes and the potential indeed vanishes, as required by the BPS condition. As the simplest example of a configuration for which a potential exists, we take $n = -n' = 1$ and obtain a result that is 16 times that for the interaction between a D0-brane and the wrapped brane. This reproduces the correct potential between wrapped branes. The same potential was analyzed in [15,16] for infinite membranes, where an ambiguity as to the correct counting of the degeneracies of membrane states was found. If one treats the constant F exactly in the Hamiltonian and disregards the effects of the boundary of the torus, one obtains a splitting between bosons and fermions in the energy levels [15]. To obtain the correct potential, one has to take into account the degeneracy of these Landau levels[3], leading to an extra factor of n. This reproduces the F^4 interaction we had before, and it also shows the appearance of a tachyon in the spectrum for small separations, in agreement with the results previously obtained in [15,16].

On compactifying further dimensions, the wrapped membrane states that we have described remain BPS saturated solutions of the appropriate SYM theory. The same arguments we present above can be carried out and the only modifications are in the structure of the second homology groups and in the value of the gravitational constant and R dependence.

Now that we have seen that we can correctly produce all of the BPS saturated states, it is interesting to investigate non-BPS saturated solutions. Recall that, in general, we have the splitting $U(N_1) \otimes U(N_2) \otimes \cdots \otimes U(N_k)$, so these states can be interpreted as bound states of wrapped membranes. Since a tachyon develops in the theory at short-distances, these states are unstable. However, the fact that they are classical solutions to the Yang-Mills equations means that they should provide the intermediate states in studies of brane-antibrane annihilation and in brane scattering with transfer of RR charge. We hope to return to these considerations in a future work.

If we compactify two more dimensions, we take $\mathcal{T} = T^4$ and have $4 + 1$-dimensional SYM on the dual torus $\widetilde{\mathcal{T}}$. Associated to this gauge theory, we now have a new quantum number, namely the instanton number for the $SU(N)$ factor, which is the second Chern class, $c_2(SU(N))$. A construction analogous to that above introduces an object that is independent of the D0-branes and membranes we had before. In the IIA picture, this must correspond to D4-branes wrapped around a 4-cycle, a description already given by [18]. Such a topological analysis of D4-brane bound states appears in [21]. Again, with $c_2(SU(N)) \in H^4(\mathcal{T}, \mathbb{Z})$, this result generalizes straightforwardly to higher compactifications.

Moreover, one may conjecture that the third Chern class should give some notion of

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[3]We thank M. Berkooz for emphasizing this point to us.

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wrapped six-branes in Type IIA compactified to four dimensions.

5. Conclusions

In this letter, we have shown that the gravitational interactions of M(atrix)-Theory correspond exactly to those of supergravity, in flat space and for all toroidal compactifications down to four flat dimensions. We have also given a numerically precise description of membranes which are wrapped on 2-cycles of tori, exactly reproducing the supergravity interactions of membranes. The behavior of these wrapped membranes is consistent with an explicit realization of T-duality in M(atrix)-Theory.

It appears that the physics of wrapped membranes in M(atrix)-Theory is completely contained in a sum over the different topological sectors of the matrix SYM describing the system. Considering that the quantization of membranes is itself a non-trivial matter, it is rather remarkable that the problem is relatively tame here. The topological properties of M(atrix)-Theory that we have uncovered should prove to be of some importance in discussing compactifications on non-trivial manifolds [22,23,24,25]. In particular, it would be very interesting if one could generalize our results to non-toroidal compactifications. We are currently working on these issues.

Conventions for M-Theory and 11-Dimensional Gravity

We would like to establish the precise conventions that produce exact agreement between M-Theory and M(atrix)-Theory. We take the coefficient of the Einstein action to be

$$S_{\text{grav}} = \frac{1}{2\kappa^2} \int d^{11}x \sqrt{g} R \tag{A.1}$$

and in linearization use the graviton normalization $(g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu})$

$$h_{\mu\nu} = \frac{1}{\sqrt{2k_{11}}} e_{\mu\nu}(k) e^{ik \cdot x}, \tag{A.2}$$

where $|e_{\mu\nu}|^2 = 1$. To calculate κ^2 we use the Schwarz formula for the M2 and M5-brane tensions [26],

$$T^{(5)} = \frac{\left(T^{(2)}\right)^2}{2\pi}, \tag{A.3}$$

and the quantization condition [27]

$$2\kappa^2 T^{(2)} T^{(5)} \in 2\pi\mathbb{Z}, \tag{A.4}$$

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so that we can express

$$\kappa^2 = \frac{2\pi^2}{\left(T^{(2)}\right)^3}.$$ (A.5)

Since the IIA string is obtained by wrapping the M2-brane around the 11th dimension,

$$2\pi r T^{(2)} = T_A = \frac{M_s^2}{4\pi},$$ (A.6)

so that we can alternatively express κ^2 in the string units of (3.1)

$$\kappa^2 = \frac{(2\pi)^5}{2} \left(\frac{r}{T_A}\right)^3.$$ (A.7)

Acknowledgements

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